Multi-user Diversity in Spectrum Sharing Systems over Fading Channels with Average Power Constraints

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Abstract

The multi-user diversity in spectrum sharing cognitive radio systems with average power constraints over fading channels is investigated. Average power constraints are imposed for both the transmit power at the secondary transmitter and the interference power received at the primary receiver in order to provide optimal power allocation for capacity maximization at the secondary system and protection at the primary system respectively. Multiple secondary and primary receivers are considered and the corresponding fading distributions for the Rayleigh and Nakagami$-m$ fading channels are derived. Based on the derived formulation of the fading distributions, the average achievable channel capacity and the outage probability experienced at the secondary system are obtained, revealing the impact of the average power constraints on optimal power allocation in multi-user diversity technique in fading environments with multiple secondary and primary receivers that share the same channel. The obtained results highlight the advantage of having on one hand more secondary receivers and on the other hand fewer primary receivers manifested as an increase in the achievable capacity.

Index Terms

Spectrum sharing, cognitive radio, multi-user diversity, average power constraints, optimal power allocation, fading channels.

I. Introduction

Cognitive radio, CR, technology has been introduced in order to mitigate the underutilization of the available radio spectrum by using the assigned spectrum in a more efficient manner [1]. In a CR system, the spectrum allocated to a licensed network (i.e. a primary system) can be used by a secondary system. This can be done via one of two types of CR systems: opportunistic spectrum access (OSA), and spectrum sharing (SS) [2]. The OSA paradigm relies on exploiting spectrum gaps that become available in the primary systems, and then are used by the secondary system while the SS paradigm relies on the coordinated sharing of a spectrum band among the primary and the secondary systems. The functionality of these two types is based on power control (PoC) mechanisms.

The PoC in OSA systems gives the cut-off level in the transmit power, above which a secondary user (SU) of the secondary system can have access to a frequency band. In SS systems, the PoC additionally satisfies the maximum allowable interference level at the primary user (PU) thereby guaranteeing reliable operation for the PU [3][4][5].

Communication among multiple SUs over fading channels raises new performance issues for CR systems that could be managed using selective transmission for one SU among others, which can achieve multi-user diversity (MUD) gain due to the time variations produced by the fading channels [6]. MUD for conventional networks has been studied extensively in order to realize the gain that can be achieved [7][8][9]. Potential MUD has recently been studied for CR systems and especially for SS systems that provide protection for PUs. In this context, it is also at interest to investigate the MUD for SS systems jointly with PoC in which an additional interference power constraint should be taken into account for protecting the PU's transmissions.

Towards this direction, in [10], the authors consider an SS system in three different scenarios: the multiple access channel, the broadcast channel and the parallel access channel. They assume secondary receivers (SU-Rx), which communicate with one secondary transmitter (SU-Tx) imposing peak transmit power and peak interference power constraints for the protection of the primary receiver (PU-Rx). Based on this model, they obtain the multi-user diversity gain (MDG) which quantity known as multi-user interference diversity (MID). They obtain numerical results for a Rayleigh fading distribution that shows the impact of MID on the ergodic throughput in relation to the SU-Rx for the three defined scenarios. In [11], the authors investigate the effects of MUD in an SS system in which an SU-Rx communicates with an SU-Tx applying peak transmit power and peak interference power constraints for the protection of the PU-Rx. Based on this model, the authors derive the ergodic capacity over Rayleigh fading channels for the high power SNR regime considering one PU-Rx and multiple PU-Rx and they provide corresponding numerical results. In [12], the authors assume an SU-Tx that serves SU-Rx in an SS system and produce interference at PU-Rx. Based on this model, the SU-Tx selects the SU-Rx with the strongest channel gain while it keeps simultaneously the peak interference power at the primary receivers (PU-Rxs) at tolerable levels. In [13], the authors investigate the SS system proposed in [11] in terms of hyper-fading communication channels for the secondary and primary links. The fading channel model is represented by a hyper Nakagami-m distribution which models a variety of propagation environments. They derive the corresponding probability density function and the cumulative density function considering peak values for the constraints on both the transmit and interference powers. In [14], the author investigates MUD in an SS system with multiple SU-Rxs and one SU-Tx over Rayleigh fading channels and derives the outage and effective capacities considering peak values for the constraints as well. The same author in [15] investigates the uplink scenario in an SS system with multiple SU-Rxs and results are obtained for Rayleigh fading channels, including the achievable bit error rate and mean capacity using an outage capacity formulation. This work also considers peak values for the power constraints.

It is clear from the above that the previous MUD investigations in SS systems have considered peak values for PoC. However, the PoC mechanism can also be implemented with average power constraints [4]. It has recently been recognized that average interference power constraints provide a more flexible power allocation policy by which the transmit power of the SU is regulated under different fading conditions and thus the achievable capacity

is increased [16]. Moreover, the average power constraints results in water-filling based optimal power allocation [4], which is beneficial in terms of capacity in low SNR regimes but not in high SNR regimes in which the transmit power tends to be fixed and the gain of this implementation negligible [17]. The aim of this paper is to analyze and investigate the capacity of SS systems that on one hand employ MUD for the selection of the SU with the best channel quality and on the other hand provide PU's protection by imposing average power constraints for both the transmit and the interference powers defining in this way the corresponding OPA policies. To the best of our knowledge, this aspect of SS systems has not investigated previously. For instance, [18] presents different OPA policies for CR systems in a multi band and multi user context considering three OPA policies: the truncated channel inversion policy, the water filling policy and the Hayes policy; however, the CR model does not involve SS aspects with PU's protection through an interference power constraint. In [19], the same authors consider an outage probability for the PU's protection which must be kept in a specific upper level while the OPA policy is taken with peak interference power constraint and thus no average values are taken into account.

The rest of this paper is organized as follows. In Section II, the system model is presented. The problem formulation for an SS system with MUD and average power constraints is formulated and solved in Section III and in Section IV, the capacity of the considered SS system is defined. In Section V, the required fading distributions for different fading channels is derived for one PU and in Section VI for multiple PUs is provided. In Section VII, the analytical and simulation results for the achievable capacity and the outage probability are presented. Finally, this paper is concluded in Section VIII.

II. System Model

We consider a spectrum sharing system that consists of a secondary system with one secondary transmitter denoted as SU-Tx and K secondary receivers denoted as SU-Rxs that utilizes a spectral band that is licensed to the primary system. The primary system is considered with multiple primary receivers denoted as PU-Rxs. The instantaneous channel power gains from the SU-Tx to the different SU-Rxs and PU-Rxs are denoted as $g_{s,i}$ and $g_{p,j}$ respectively with $i \in [1,..,K]$ and $j \in [1,..,L]$. The model of the considered SS system is depicted in Fig.1. All channel gains are assumed to be independent and identically distributed (i.i.d.) exponential random variables with unit means in independent Nakagami-m fading channels [20] and independent additive white Gaussian noise (AWGN) with random variables denoted as $n_{s,i}$ and $n_{p,j}$ for the primary and secondary links respectively with means zeros and variances N_0 [21].

The SU-Tx regulates its transmit power through the power control mechanism that provides transmission with power constraints at both secondary and primary links in order to satisfy the requirements for transmission and protection at the SU-Rxs and the PU-Rxs simultaneously in the considered SS system. To this end, we assume that perfect channel state information (CSI) is available at the SU-Tx from the SU-Rxs and the PU-Rxs through a feedback channel [22]. It is assumed that the knowledge of PU-Rxs CSI can be acquired with different alternatives as discussed in [23]. Besides, the SU-Tx exploits the multi-user diversity selection strategy by which it is able to select for transmitting information among multiple SU-Rxs to the one with the best received signal-to-noise ratio

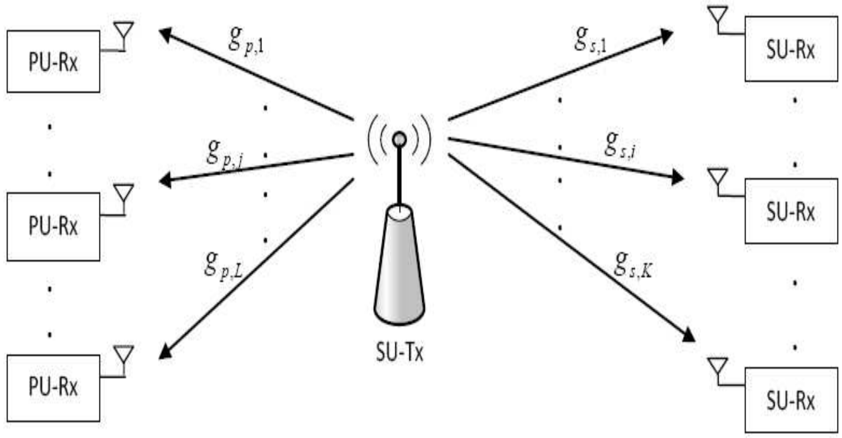

Fig. 1. System model of an SS system with multiple SU-Rxs and PU-Rxs

(SNR). For the considered SS system, the received SNR at the $i-th$ SU-Rx is given as $\gamma_{s,i} = P_t g_{s,i}/n_{s,i}^2$. We assume that the power constraints at the PoC of the SU-Tx are applied for keeping the transmit power budget at the secondary links under a predefined level assuming an average value P_{av} as well as for keeping the interference power at the primary links at a tolerable level, assuming an average value I_{av} [4]. Throughout this paper we will consider these average values for the power constraints that can provide the optimal power allocation policy for the secondary system. It has been recognized that such a transmission policy is beneficial for the conventional systems especially in the low SNR regime [17]; however it has not been investigated for SS CR systems that employ also MUD, as has been discussed the literature in [11][13].

III. Spectrum Sharing with Multi-User Diversity and Average Power Constraints

A. Multi-user Diversity in Spectrum Sharing

We assume that the SU-Tx employ MUD by which the SU-Tx selects the SU-Rx that has the best channel quality among all SU-Rxs. Thus, the received SNR of the selected SU-Rx $\gamma_{s,max}$ is obtained as follows [24]:

$$\gamma_{s,max} = \max_{1\leq i\leq K} \gamma_{s,i} \tag{1}$$

with probability density function (PDF) obtained as follows:

$$f_{\gamma_{s,max}}(x) = K f_{\gamma_{s,i}}(x) F_{\gamma_{s,i}}(x)^{K-1} \tag{2}$$

where $f_{\gamma_{s,i}}(x)$ and $F_{\gamma_{s,i}}(x)$ are the PDF and the cumulative distribution function (CDF) of the received SNR $\gamma_{s,i}$ at the $i-th$ SU-Rx respectively. The overall average achievable capacity at the secondary system (i.e. SU-Tx to

SU-Rx) is obtained as follows:

$$\begin{aligned} C_s &= E[log(1+\gamma_{s,max})] \\ &= \int_0^\infty log(1+\gamma_{s,max}) f_{\gamma_{s,max}}(x)dx \end{aligned} \tag{3}$$

As is shown also in [11] and [13], since the density function $f_{\gamma_{s,max}}(x)$ in (3) depends on the received SNRs $\gamma_{s,i} = P_t g_{s,i}/n_{s,i}^2$ at the SU-Rxs, the capacity of the secondary system in (3) depends on the transmit power of the SU-Tx. Again, against of using the peak values for the transmission policy of the SU-Tx, we consider average values for the power constraints. As will be presented below, the analysis is changed due to the dependencies among the channel gains $g_{s,i}$ and $g_{p,j}$ at the secondary and primary links respectively that are given rise by the obtained power transmission policies [25].

B. Spectrum Sharing with Multi-user Diversity Under Average Power Constraints

Considering the PoC in an SS system that follows MUD as described above, we formulate the objective of system with the following optimization problem. The problem under consideration is to maximize the capacity of the secondary system subject to average transmit power constraint P_{av} for the SU-Tx as well as the average interference power constraint I_{av} at the PU-Rx. Such a related maximization problem has been discussed in [4] and [16] that we describe and solve this problem below.

Problem: The aforementioned capacity maximization problem for the considered SS system is defined as follows

$$\max_{P_t} \qquad E[log_2(1+\frac{P_t g_{s,i}}{n_{s,i}^2})] \tag{4}$$

$$s.t. \qquad E[P_t] \leq P_{av} \tag{5}$$

$$E[g_{p,j}P_t] \leq I_{av} \tag{6}$$

Solution: We consider the dual problem where its duality gap is zero for the convex optimization problem addressed here which means the solution of the dual problem is equivalent to one of the original problem [26]. The dual Lagrangian function is given as follows:

$$\begin{aligned} L(P_t,\lambda,\nu) &= E[log(1+\frac{P_t}{g_{s,i}}n_{s,i}^2)] \\ &+ \lambda(I_{av} - E[g_{p,j}P_t]) + \nu(P_{av} - E(P_t)) \end{aligned} \tag{7}$$

where λ and ν are Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) conditions yield the following equations:

$$n_{s,i}^2 g_{s,i}^{-1} + {P_t}^{-1} - \lambda g_{p,j} - \nu = 0 \tag{8}$$

$$\lambda(I_{av} - E[g_{p,j}P_t]) = 0 \tag{9}$$

$$\nu(P_{av} - E(P_t)) = 0 \tag{10}$$

where $\lambda \geq 0$ and $\nu \geq 0$, which yield the solution of the specific dual problem results via the following equation:

$$P_t = \frac{1}{\lambda g_{p,j} + \nu} - \frac{n_{s,i}^2}{g_{s,i}}, \qquad \forall i,j \tag{11}$$

The parameters λ and ν are obtained by substituting (11) into (9) and (10). The solution of this problem assumes the average values for both the transmit and interference power constraints in SS systems, which has been discussed in [4] although was not proved. In fact, (11) encapsulates the OPA policies as expressed by the optimal values λ and ν that are being regulated over the constraint on the primary links i.e. interference power constraint I_{av} and the constraint on the secondary links i.e. transmit power constraint P_{av} respectively. In particular, when strict inequality holds for both constraints the solution can be obtained alternatively using the equations (9) and (10) as noted also in [4]. This property has been also shown in [27] where the calculation of both and coefficients is unnecessary when the transmit power is constrained using peak values for the corresponding regulatory constraints i.e. I_{pk} and P_{pk} respectively. We present below the strategy for obtaining the parameters λ and ν via equations (9) and (10).

C. Average Interference Power Constraint I_{av}

We assume now that in the previous problem the transmit power of the SU-Tx is obtained from the average interference power constraint I_{av} and thus based on equation (11) can be computed as follows [4]

$$P_t = \left[\frac{1}{\lambda g_{p,j}} - \frac{n_{s,i}^2}{g_{s,i}}\right], \quad if \quad g_{s,i}/n_{s,i}^2 \geq \lambda g_{p,j} \tag{12}$$

and the λ coefficient that represents the cut-off level of the received SNR at the SU-Rx can be calculated by the substitution of (12) into (9) using the following equation

$$E\left[\frac{1}{\lambda} - n_{s,i}^2 \frac{g_{p,j}}{g_{s,i}}\right] = I_{av} \tag{13}$$

It is evident from (13) that the OPA policy follows the water-filling principles and more specifically, for each SU-Rx, the water level is determined by the term $g_{s,i}/g_{p,j}$ as has also been indicated in [4] and [25]. Due to that term, the fading distribution for handling (13) is changed as we will discuss below in order to calculate the λ coefficient.

D. Average Transmit Power Constraint P_{av}

We assume now the case in which the transmit power of SU-Tx is obtained via the average transmit power constraint P_{av} and thus, following the same procedure as above the transmit power of the SU-Tx is obtained as follows:

$$P_t = \left[\frac{1}{\nu} - \frac{n_{s,i}^2}{g_{s,i}}\right], \quad if \quad g_{s,i}/n_{s,i}^2 \geq \nu \tag{14}$$

and the ν coefficient that represents the cut-off level of the received SNR at the SU-Rx can be calculated by the substitution of (14) into (10) using the following equation

$$E\left[\frac{1}{\nu} - \frac{n_{s,i}^2}{g_{s,i}}\right] = P_{av} \tag{15}$$

Obviously, the transmission policy in (15) is the well-known water-filling power allocation technique [17] without dependencies on the channel gain of the primary link as in (13). This means that the solution of the problem is conventional water filling; however the transmit power is constrained by the average interference power constraint I_{av} as presented above for protecting the primary users.

IV. Capacity in Spectrum Sharing with Multi-User Diversity and Average Power Constraints

The MUD in the considered SS system provides the selection of the SU-Rx with the best channel quality i.e. that which shows the maximum instantaneous SNR $\gamma_{s,max}$ as expressed in (1). Taking into account that the $\gamma_{s,max} = P_t g_{s,max}/n_{s,i}^2$ is achieved for the maximum channel power gain $g_{s,max}$ and that the transmit power P_t is given by the corresponding OPA policy, we can obtain the capacity of the SS system with MUD and OPA from (4) in terms of the interference power I_{av} and transmit power P_{av} using the equations (12) and (14) respectively.

A. Average Interference Power Constraint I_{av}

We consider the case of the average interference power constraint I_{av} that has resulted in OPA in (12) and we define as $Z = g_{s,max}$ and $X = g_{s,max}/g_{p,j}, \forall j$; thus we have the following formulation for the average achievable capacity at the SU-Tx when MUD is applied as well:

$$\begin{aligned} C_s &= \int_0^\infty log(1 + P_t \frac{z}{n_{s,i}^2})\ f_{\gamma_{s,max}}(z)\, dz \\ &= \int_\lambda^\infty log(\frac{z}{\lambda g_{p,j} n_{s,i}^2})\ f_{\gamma_{s,max}}(z)\, dz \\ &= \int_\lambda^\infty log(\frac{x}{\lambda n_{s,i}^2})\ f_{\gamma_{s,max}}(x)\, dx \end{aligned} \tag{16}$$

Obviously, by substituting the $X = g_{s,max}/g_{p,j}$ in (16) the water-filling algorithm expressed by (13) leads to the inference that the water level is proportional to the channel gain on the primary link denoted by $g_{p,j}$ (see also [25]). In this case, the achievable capacity at the secondary link in (16) can be obtained from the PDF of this term i.e. $g_{s,max}/g_{p,j}$ as accomplished also in [4] and [16] but without MUD. Obviously, the received SNR $\gamma_{s,i}$ at the $i-th$ SU-Rx is now expressed in relation to this term. Taking into account this aspect of the fading environment in SS systems, we derive in the next section the CDF $F_{\gamma_{s,i}}(x)$ of the received SNR $\gamma_{s,i}$ at the $i-th$ SU-Rx using equation (3) for Rayleigh and Nakagami$-m$ fading channels. They will be used in the sequel in order to calculate the PDF of the received SNR $\gamma_{s,max}$ of the selected SU-Rx using (2) and thus finally the average capacity from (17) respectively. In addition to that, we consider multiple PU-Rxs, which will also require a new formulation in terms of the fading distributions.

B. Average Transmit Power Constraint P_{av}

We consider the case of the average transmit power constraint P_{av} that has resulted in OPA in (14) and we keep the notation $Z = g_{s,max}$; thus we have the following formulation for the average achievable capacity at the SU-Tx when MUD is applied as well:

$$\begin{aligned} C_s &= \int_0^\infty log(1 + P_t \frac{z}{n_{s,i}^2})\ f_{\gamma_{s,max}}(z)\, dz \\ &= \int_\nu^\infty log(\frac{z}{\nu n_{s,i}^2})\ f_{\gamma_{s,max}}(z)\, dz \end{aligned} \tag{17}$$

Obviously, both formulation for the capacities in (16) and (17) represents water-filling based solutions. In (16) the dependency on the channel gain at the primary link is reasonable since the transmit power is regulated by the average

power constraint on the secondary link i.e. P_{av}. Equation (17) can be obtained from the PDF of the term $g_{s,max}$ only as accomplished also in [22] and [28] in the case of MUD without considering an interference power constraint. Making both calculation we can derive the achievable capacity of the SU-Tx over fading channels for specific average values of P_{av} and I_{av}. However, for the aforementioned calculations the fading distributions are required and in particular the case of average interference power constraint I_{av} requires some additional manipulation since the parameter of interest is proportional to the channel gain at the primary link denoted as $g_{p,j}$, while in case of average transmit power constraint P_{av} the fading distribution is identical to the conventional one. More specific, in the case of the average transmit power there is no need for a special formulation since this solution represents the conventional water-filling algorithm without dependency on interference channel gains $g_{p,j}$. Henceforth, we derive below the required fading distributions in the case of an average interference power constraint. Based on these distribution we are able to calculate the capacity and the outage probability as well [21]

$$\begin{aligned} P_{out} &= P[g \leq g^*] \\ &= \int_0^{g^*} f_{\gamma_{s,max}}(x)\, dx \end{aligned} \tag{18}$$

where g^* is the optimal cutoff level of the channel power gain i.e. no data are transmitted below this power level in which depends on in the case of the average interference power constraint I_{av} it is considered as the optimal value λ^* obtained by (13), and in the case of the average transmit power constraint P_{av} as the optimal ν^* as obtained by (15).

V. Fading Distributions for Multi-user Diversity and Average Interference Power Constraint

A. Rayleigh distribution

Here, we assume that the channels gains $g_{s,i}$ and $g_{p,j}$ are i.i.d. Rayleigh random variables $\forall i, j$. For notational brevity, we will denote the term $g_{s,max}/g_{p,j}$ as g_s/g_p and as before we will substitute $X = g_s/g_p$ so that the PDF of the received SNR at the SU-Tx is obtained following an identical procedure as in [11] as follows

$$\begin{aligned} f(x) &= \int_0^{\infty} z e^{-xz} e^{-z}\, dz \\ &= -\frac{e^{-(1+x)z}(1+z+xz)}{(1+x)^2}\, |_0^{\infty} = \frac{1}{(1+x)^2} \end{aligned} \tag{19}$$

which is equal to the expression presented in [25]. The CDF of the PDF in (19) is obtained as follows:

$$F(x) = 1 - \frac{1}{1+x} \tag{20}$$

Substituting (19) and (20) into (2), we can derive the PDF $f_{\gamma_{s,max}}(x)$ of the maximum received SNR $\gamma_{s,max}$ of the selected SU-Rx, and using (13) the coefficient λ should follow the following constraint:

$$\int_{\lambda^*}^{\infty} (\frac{1}{\lambda^*} - n_{s,i}^2 x) f_{\gamma_{s,max}}(x)\, dx = I_{av} \tag{21}$$

The corresponding average capacity and outage probability are obtained by (16) and (18) respectively using the derived $f_{\gamma_{s,max}}(x)$ for the optimal value λ^* which follows the OPA in (21).

B. Nakagami$-m$ distribution

We now assume that the channels gains $g_{s,i}$ and $g_{p,j}$ are i.i.d. Nakagami$-m$ random variables $\forall i, j$ and thus follow the following Nakagami$-m$ distribution for a specific channel gain $Z = z$:

$$f(z) = \frac{m^m z^(m-1)}{\Gamma(m)} e^{(-mz)}, \qquad z \geq 0 \tag{22}$$

where m represents the shape factor under which the ratio of the line-of-sight (LoS) to the multi-path component is realized [20]. Assuming that both channels gains $g_{s,i}$ and $g_{p,j}$ have instantaneously the same fading fluctuations i.e. $m_s = m_p = m$, the PDF of the term $X = g_s/g_p$ is obtained as follows [25]:

$$f(x) = \frac{x^{m-1}}{B(m,m)(x+1)^{2m}}, \qquad x \geq 0 \tag{23}$$

After some mathematical manipulation, the CDF of the PDF in (24) is obtained as follows:

$$F_{g_s/g_p}(x) = \frac{1}{B(m,m)} \frac{x^m}{m} {}_2F_1(m, 2m; 1+m; -x) \tag{24}$$

where ${}_2F_1(a, b; c; y)$ is the Gauss hyper-geometric function which is a special function of the hyper-geometric series [29]. Substituting (24) and (23) into (2), we can derive the PDF of the received SNR $\gamma_{s,max}$ of the selected SU-Rx and finally the optimal coefficient λ^* which will be used below for the calculation of the corresponding average capacity and outage probability as mentioned above.

VI. Multiple PUs Scenario with Average Interference Power Constraint

We assume now that the spectrum is shared among L PUs (i.e. PU-Rx) as mentioned in the system model. This design has been accomplished in [11] and [25] for the Rayleigh fading case and where the corresponding performance degradation in terms of capacity is examined. In this section, we will present additionally how the multiple PUs scenario can be considered in the Nakagami$-m$ fading case that has not been presented in [13] which presents the case with one PU only. Both Rayleigh and Nakagami$-m$ fading cases will be used for the capacity calculation with OPA in a MUD context with multiple PUs.

A. Rayleigh fading

Based on (19) and taking into account the analysis presented in [25] the PDF of the term g_s/g_p in Rayleigh fading channels with multiple PUs is given as follows:

$$f(x) = L \sum_{0}^{L-1} (-1)^h \binom{L-1}{h} \frac{1}{(1+x+h)^2} \tag{25}$$

After some mathematical manipulation, the CDF of the PDF in (25) is obtained as follows:

$$F(x) = L \sum_{0}^{L-1} (-1)^h \binom{L-1}{h} \frac{1}{1+h} - \frac{1}{1+x+h} \tag{26}$$

Next, we follow the same procedure in order to derive the PDF of the term g_s/g_p in Nakagami$-m$ fading channels with multiple PUs.

B. Nakagami$-m$ fading

We assume the interference channel gains $g_{p,j}$ between the SU-Tx and each $j = 1, \therefore, L$ PU-Rx are i.i.d. unit mean Nakagami random variables. We assume also that each transmit channel gain $g_{s,i}$ with $i = 1, \therefore, K$ is also independent of all the $g_{p,j}$. We define now a variable for taking the maximum value of the interference channel denoted as g_p that can be obtained as follows:

$$g_p = \max_j g_{p,j}, \qquad j = 1, \therefore, L \tag{27}$$

The equation (27) represents the maximum value of all channel gains $g_{p,j}$ between the SU-Tx and the PU-Rxs. The CDF of this value g_p is obtained as follows:

$$F_{g_p}(x) = \prod_{j=1}^{L} F_{g_{p,j}}(x) \tag{28}$$

where $F_{g_{p,j}}(x)$ is the CDF of the Nakagami$-m$ fading distribution which can be derived from the integration of (22) as follows:

$$\begin{aligned} F_{g_{p,j}}(x) &= \int_0^x \frac{m^m x^m - 1}{\Gamma(m)} e^{-mx}\, dx \\ &= \frac{1}{\Gamma(m)}(1 - \Gamma(m, mx)), \forall j \end{aligned} \tag{29}$$

Therefore, substituting (29) into (28) the following is applied for the CDF of g_p:

$$\begin{aligned} F_{g_p}(x) &= \prod_{j=1}^{L} \frac{1}{\Gamma(m)}(1 - \Gamma(m, mx)) \\ &= \frac{1}{\Gamma(m)}(1 - \Gamma(m, mx))^L \end{aligned} \tag{30}$$

We can now obtain the PDF of g_p by differentiating (30) as follows

$$\begin{aligned} f_{g_p}(x) = \frac{d}{dx} F_{g_p}(x) \\ = \quad m^m e^{-mx} x^{m-1} \frac{L}{\Gamma(m)^L}(1 - \Gamma(m, mx))^{L-1} \end{aligned} \tag{31}$$

We now proceed with the formulation of the PDF of the proportional channel gain used in SS systems i.e. g_s/g_p. Following the same procedure as in [11] and [25], the PDF of the g_s/g_p is defined in general as follows

$$f_{g_s/g_p}(x) = \int_0^x f_{g_s}(\frac{xy}{1+x}) f_{g_p}(\frac{y}{1+x}) \frac{y}{(1+x)^2}\, dy \tag{32}$$

where $f_{g_s}(.)$ is the PDF at the secondary link obtained by (22) for the Nakagami$-m$ distribution and $f_{g_p}(.)$ is the PDF at the primary link with the maximum channel gain value as obtained in (31). Thus, the (32) becomes:

$$\begin{aligned}
& f_{g_s/g_p}(x) = \int_0^\infty \frac{m^m}{\Gamma(m)}(\frac{x^y}{1+x})^{m-1}e^{-m\frac{xy}{1+x}}L\frac{m^m}{\Gamma(m)} \\
& (\frac{y}{1+x})^{m-1}e^{-m\frac{y}{1+x}}(\frac{1-\Gamma(m,m\frac{y}{1+x})}{\Gamma(m)})^{L-1} \\
& \frac{y}{(1+x)^2}\,dy = L\frac{m^{2m}}{\Gamma(m)^{2+1}}\frac{x^{m-1}}{(1+x)^{2m}} \\
& \int_0^\infty y^{2m-1}e^{-my}(1-\Gamma(m,m\frac{y}{1+x}))^{L-1}\,dy \\
= \; & L\frac{m^{2m}}{\Gamma(m)^{2+1}}\frac{x^{m-1}}{(1+x)^{2m}}\int_0^\infty y^{2m-1}e^{-my} \\
& \sum_{h=0}^{L-1}(-1)^h\binom{L-1}{h}\Gamma(m,m\frac{m}{m\frac{hy}{1+x}})\,dy \qquad (33) \\
= \; & L\frac{m^{2m}}{\Gamma(m)^{2+1}}\frac{x^{m-1}}{(1+x)^{2m}}\sum_{h=0}^{L-1}(-1)^h\binom{L-1}{h} \\
& \int_0^\infty y^{2m-1}e^{-my}\Gamma(m,m\frac{m}{m\frac{hy}{1+x}})\,dy \qquad (34)
\end{aligned}$$

The integral in (34) is solved by using the equation (6.455.1) of the tables in [30] leading to

$$\begin{aligned}
& f_{g_s/g_p}(x) = L\frac{\Gamma(3m)}{\Gamma(m)^{L+1}}\frac{x^{m-1}}{2m}\sum_{h=0}^{L-1}(-1)^h\binom{L-1}{h} \\
& \frac{h^m}{(x+h+1)^{3m}}\,{}_2F_1(1,3m;2m+1;\frac{x+1}{x+h+1}) \qquad (35)
\end{aligned}$$

where ${}_2F_1$ is a hyper-geometric function. Substituting $m=1$ (i.e. the Rayleigh distribution) into (35), we have

$$\begin{aligned}
P1: \qquad & f_{g_s/g_p}(x) = L\frac{\Gamma(3)}{\Gamma(1)^{L+1}}\frac{x^0}{2}\sum_{h=0}^{L-1}(-1)^h\binom{L-1}{h} \\
& \frac{h}{(x+h+1)^3}\,{}_2F_1(1,3,3;\frac{x+1}{x+h+1})
\end{aligned}$$

where the *hypergeometric* function in (P1) for the specific values gives the following equation

$$P2: {}_2F_1(1,3,3;\frac{x+1}{x+h+1}) = \frac{1}{1-\frac{x+1}{x+h+1}} = \frac{x+h+1}{h}$$

Substituting (P2) into (P1) then we have the following expression for the PDF for $m=1$:

$$P3: f_{g_s/g_p}(x) = L\sum_{h=0}^{L-1}(-1)^h\binom{L-1}{h}\frac{1}{(x+h+1)^2}$$

which is the same as (25) [25]. In order to obtain the corresponding CDF for the Nakagami$-m$ distribution we can integrate the equation (35) substituting $m = 2$ as follows:

$$
\begin{aligned}
& F_{g_s/g_p}(x) = \int_0^x L \frac{\Gamma(6)}{\Gamma(2)^{L+1}} \frac{x}{4} \sum_{h=0}^{L-1} (-1)^h \binom{L-1}{h} \\
& \frac{h^2}{(x+h+1)^6} {}_2F_1(1, 6; 5; \frac{x+1}{x+h+1})\, dx \\
& = \int_0^x L \frac{\Gamma(6)}{\Gamma(2)^{L+1}} \frac{x}{4} \sum_{h=0}^{L-1} (-1)^h \binom{L-1}{h} \\
& \frac{h^2}{(x+h+1)^6} \frac{(1+h+x)(1+5h+x)}{5h^2}\, dx \\
& = \frac{L\Gamma(6)}{\Gamma(2)^{L+1} 20} \sum_{h=0}^{L-1} (-1)^h \binom{L-1}{h} \int_0^x x \frac{(1+5h+x)}{(x+h+1)^5}\, dx \\
& = \frac{L}{B(2,2)} \sum_{h=0}^{L-1} (-1)^h \binom{L-1}{h} (\frac{1+3h^2+4h}{6(1+h)^4} \\
& - \frac{1+3h^2+4x+3x^2+4h(1+3x)}{6(1+h+x)^4})
\end{aligned}
\tag{36}
$$

Notably, the use of a one-to-one mapping between the Ricean factor (i.e. the ratio between the power in the direct path i.e. the LoS component and the power in the other scattered paths [31]) and the Nakagami fading parameter allows also Ricean channels to be well approximated by Nakagami$-m$ channels where the relation between them for $m = 2$ gives a Ricean factor of 2.4312. Substituting (36) and (35) for $m = 2$ into (2), we can derive the PDF of the received SNR $\gamma_{s,max}$ of the selected SU-Rx when MUD is employed in the multiple PU-Rx context of SS systems in the specific fading environment. Using equation (13) the optimal coefficient λ^* of the OPA can be obtained and finally the corresponding average capacity and outage probability using equations (16) and (18) respectively.

VII. Simulation Results and Discussion

In this section, we describe and discuss simulation results derived from the above formulation of the corresponding water-filling algorithms and the fading distributions. We provide results for the achievable average capacity versus both the average transmit power P_{av} and average interference power I_{av} and we also present the corresponding outage probability P_{out}. Fig.2 shows the average capacity versus the average interference power constraint I_{av} with different numbers of SU-Rxs K and PU-Rxs L in the case of Rayleigh fading. It is evident that on one hand an increase in the number of SU-Rxs results in an increase in the capacity and on the other hand an increase in the number of PU-Rxs results in a decrease in capacity. This is expected since the probability that the SU-Tx will find an SU-Rx with the best SNR condition using MUD technique is increased and the probability that the SU-Tx will find a PU-Rx in which the interference level will be reached is increased, leading to capacity increase and decrease respectively. Fig.3 depicts the corresponding outage probability P_{out} of the implementation scenarios presented in

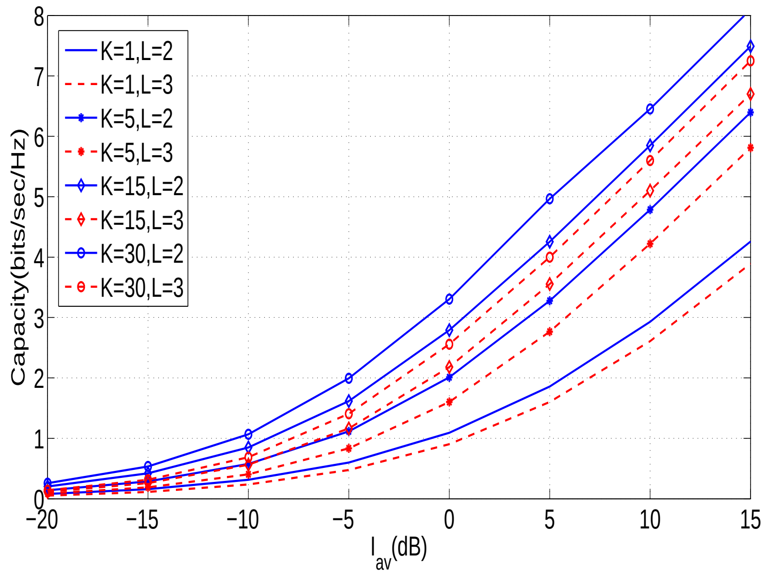

Fig. 2. Capacity vs. average interference power constraint I_{av} with different numbers of SU-Rxs K and PU-Rxs L

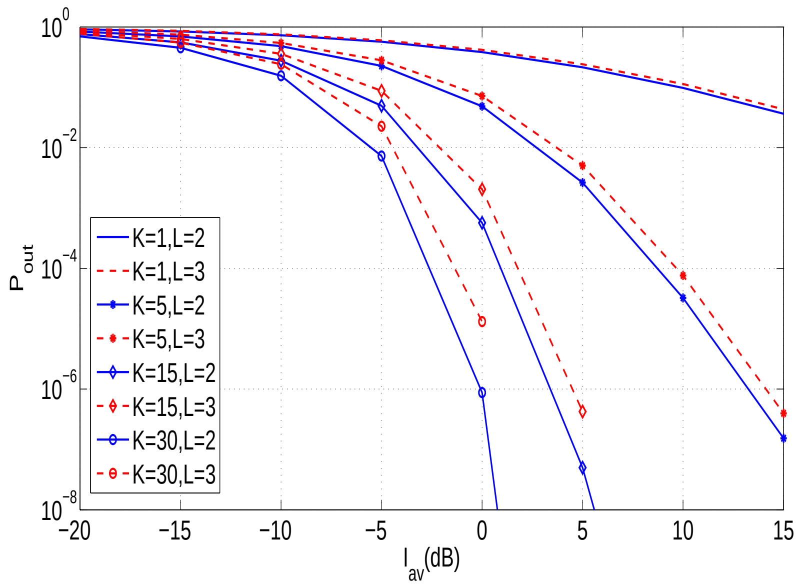

Fig. 3. Outage probability P_{out} vs. average interference power constraint I_{av} for different numbers of SU-Rxs K and PU-Rxs L

Fig.2. Obviously, the outage probability is decreasing when the number of SU-Rxs increases and increase when the PU-Rxs increases.

Fig. 4 shows the average capacity versus the average transmit power constraint P_{av} for $K = 5$ SU-Rxs and different numbers of L PU-Rx which are protected by imposing different average interference power constraint,

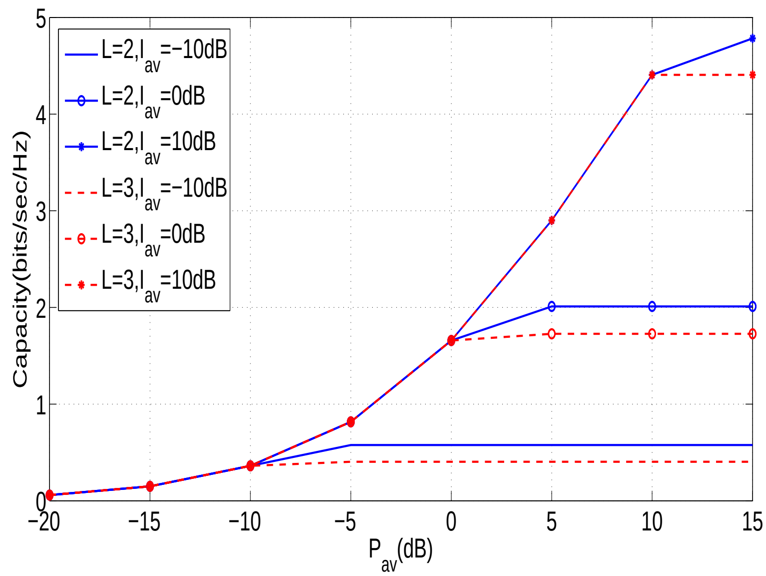

Fig. 4. Capacity vs. average transmit power constraint P_{av} for $K = 5$ SU-Rxs, different values of L, and imposing different average interference power constraints I_{av}

I_{av}. We can see that although the interference power constraint truncates the capacity, it increases when the number of PU-Rxs decreases. For example when the number of PU-Rxs is equal to $L = 2$ the capacity is truncated at a higher level than when $L = 3$ and thus even in cases of interference power constraints the capacity can be increased when the number of PU-Rxs is small. Notably, the capacity increase becomes more significant for looser constraints i.e. $I_{av} = 10dB$, and less so for tight ones e.g. $I_{av} = -10dB$. Fig.5 represents the corresponding outage probability P_{out} of the implementation scenario presented in Fig.4 where the advantages of having fewer PU-Rxs are obvious since the probability that the SU-Tx faces outage is decreased in this case.

Fig. 6 shows the achievable capacity versus the average transmit power constraint P_{av} for $K = 1$ SU-Rx and $L = 2$ PU-Rxs imposing different average interference power constraints I_{av} with Nakagami$-m$ fading with $m = 1$ (i.e. Rayleigh) and $m = 2$ (i.e. Ricean factor equal to 2.4312) highlighted with solid and dashed lines respectively. Although the capacity decreases for low values of average transmit power P_{av} in the case of $m = 2$, the opposite is true for high values of average transmit power; a similar phenomenon was observed in [13]. In high power regions the hypergeometric function in (35) is decreasing as $x \to \infty$. Thus, it can be inferred that in high power regions the SNR at the SU-Rx is not scattered significantly and thus the average power constraint results in a slight increase in capacity in these power regions. On the other hand, in cases of tight average interference power constraints i.e. $I_{av} = -10dB$ and $I_{av} = 0dB$ the truncation in capacity retains the decrease in capacity that happens in low power regions. Fig. 7 depicts the corresponding outage probability where the same outcomes are derived however in terms of outage over the different fading channels.

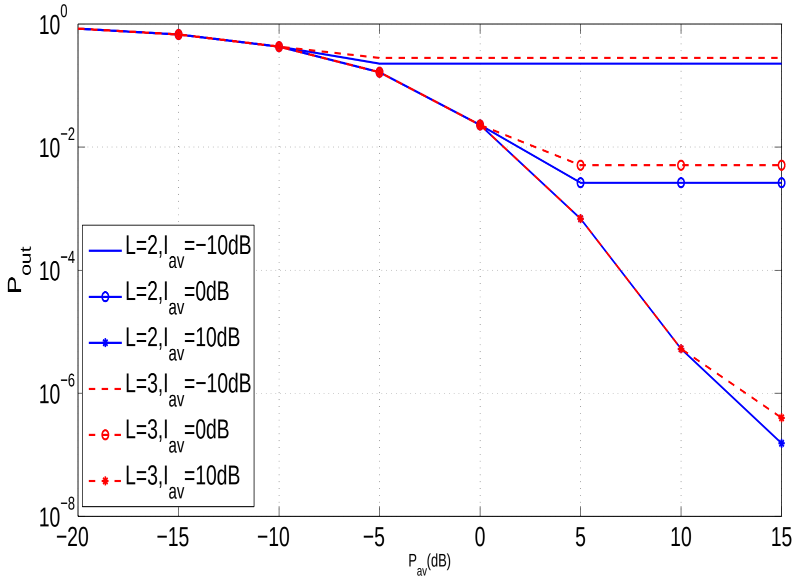

Fig. 5. Outage probability P_{out} vs. average transmit power constraint P_{av} for $K = 5$ SU-Rxs, different values of L, and imposing different average interference power constraints I_{av}

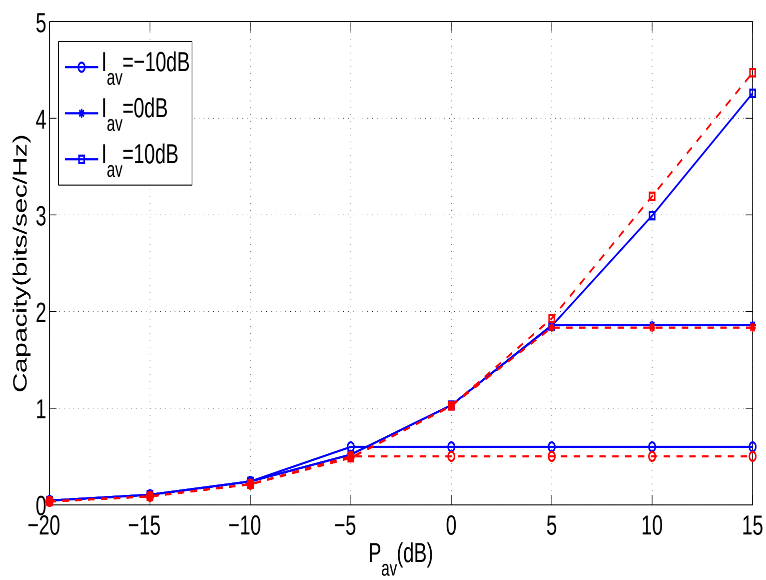

Fig. 6. Capacity vs. average transmit power constraint P_{av} for $K = 1$ SU-Rx, $L = 2$, and imposing different average interference power constraints I_{av} with $m = 1$ (solid lines) and $m = 2$ (dashed lines) for the Nakagami$-m$ distribution

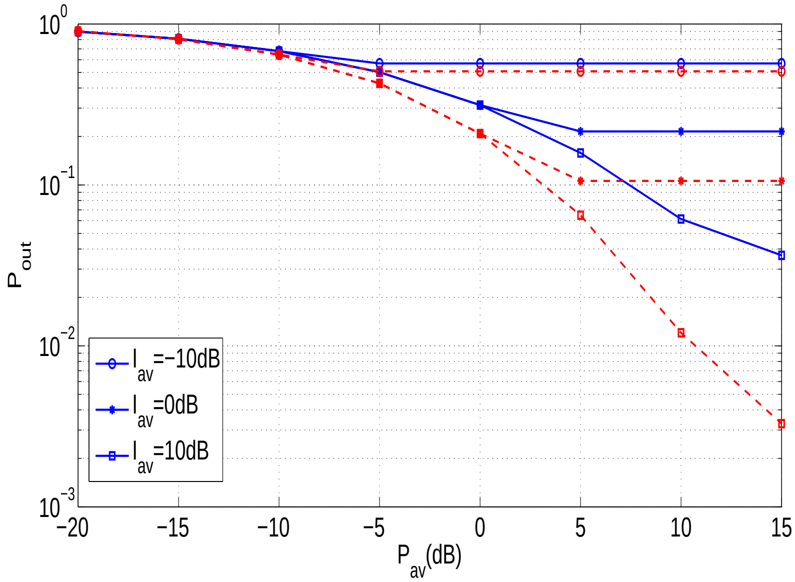

Fig. 7. Outage probability vs. average transmit power constraint P_{av} for $K = 1$ SU-Rxs, $L = 2$, and imposing different average interference power constraints I_{av} with $m = 1$ (solid lines) and $m = 2$ (dashed lines) for the Nakagami$-m$ distribution

VIII. Summary and Conclusion

In this paper, we have investigated the cognitive radio paradigm when multiple SUs and PUs share the same channel. For selecting the SU with the best channel conditions, the MUD technique has been considered in which average power constraints are imposed on the transmit power of the SU for providing optimal power allocation on the secondary links and PU protection on the primary links over fading channels. Rayleigh and Nakagami$-m$ fading were considered deriving the corresponding PDFs and CDFs that were required for calculating the achievable average capacities and the outage probabilities. Simulation results were provided in order to examine the effects of multiple SUs and PUs when MUD and optimal power allocation are employed in SS systems. It has been inferred that even in cases of tight interference power constraints increasing the number of PUs can enhance the capacity and the probability of outage as highlighted from the obtained results for different numbers of PUs. Furthermore, it has been observed that the fading environment in high power regions will give a slight increase in capacity and thus it does not have a significant impact on the achievable capacity and consequently on probability of outage.

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